Launch Vehicle Performance Estimation:

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Precise determination of launch vehicle performance typically requires the use of three- or six-degree-of-freedom simulations, via established codes such as POST or OTIS. This is an often-tedious process requiring extensive technical data on the launch vehicle, payload, and mission constraints. If this data is not available, or if high precision is not available, the use of high-fidelity 3- or 6-DOF simulations represents a waste of effort.

In many cases, a very simple and fast method of approximately determining the performance of a launch vehicle would be preferable. Examples include:

Establishing the performance trade space for a new launch vehicle
Surveying existing launch vehicles for suitability to a proposed mission
Determining the effects of mission changes on launcher performance
Preliminary evaluation of proposed launch vehicle upgrades.

Put simply, it shouldn't take more than a few minutes to estimate the performance increase associated with, e.g., a new upper-stage engine.

A potentially useful technique for such analysis was developed by George E. Townsend of the Martin Marietta corporation in 1962, and published in chapter VI of the Martin-Marietta "Design Guide to Orbital Flight". Subsequent changes in launch vehicle design practice have limited the utility of this technique, but a simple refinement is identified to address the major issues. We will start with the basic question of whether a particular launch vehicle can deliver a particular payload to a particular orbit.

The Townsend technique begins by assuming that all space launches consist of a direct ascent to a low circular parking orbit, followed by a series of on-orbit maneuvers to the final destination orbit. In fact, many launch vehicles fly only a direct-ascent trajectory, even to a high or non-circular orbit. However, an observation of these trajectories almost invariably finds the launch vehicle, at an altitude of a few hundred kilometers, accelerating almost horizontally through the local circular orbit velocity. One may simplify the problem by treating this as an instantaneous "parking orbit", reached by direct ascent, and with all subsequent powered flight treated as an "on-orbit maneuver".

On-orbit maneuvers in the general case can represent a very complex problem, which here is greatly simplified by the fact that virtually all launch vehicle propulsion systems - even relatively low-thrust upper stages - have sufficient acceleration that an impulsive-burn approximation can be used. Determination of an optimal, impulsive two- or three-burn transfer is an elementary exercise in orbital mechanics. Launch vehicle operations may face constraints (e.g. engine restart or upper-stage coast duration limits) that prevent the use of truly optimal trajectories, but it is rarely difficult to determine (estimate?) the delta-V required for on-orbit operations.

From this, the propellant requirement for on-orbit maneuvering can be determined, and bookmarked as payload for the direct-ascent portion of the trajectory. The launch vehicle performance problem is then reduced to determining whether or not the remaining launch vehicle propellant will suffice to deliver the total direct-ascent payload (i.e. sum of actual payload, upper-stage dry mass, and orbital maneuvering propellant) to the parking orbit.

This is the most complex part of the problem, as the direct-ascent trajectory requires simultaneously accelerating to parking orbit velocity and climbing to parking orbit altitude while overcoming gravity, drag, engine back pressure, and steering losses. Traditionally, all of these except parking orbit velocity or specific energy are lumped together into a generic "delta-V penalty" term, typically of magnitude ~2000 m/s and strongly dependant on launch vehicle and trajectory design.

Townsend determined that for most launch vehicles, this penalty term can be fairly accurately estimated as a simple function of direct-ascent flight time only. Based on a study of ~100 launch vehicle configurations in 1962, with takeoff thrust:weight ratios ranging from 1.2 to 2.0 and specific impulse from 250-430 seconds, the simple penalty estimation function is:

1. $\Delta V_{tot} = V^* + 1.5E\text{-}3\ T_a^2 + 8.82E\text{-}2\ T_a + 1036$ m/s

where V^* = sqrt(Vcirc^2 + 2gH(R/(R+H))^2)
T_a = ascent time, s

This penalty estimate is referenced to parking orbit energy; slightly more accurate results can be obtained by referencing to parking orbit circular velocity and using a penalty function of both altitude and ascent time. A fit to Townsend's graphical results gives a penalty function of:

2. $\Delta V_{pen} = K_1 + K_2\ T_a$

where $K_1 = 662.1 + 1.602\ H_p + 1.224E\text{-}3\ H_p^2$
$K_2 = 1.7871 - 9.687E\text{-}4\ H_p$
H_p = Parking orbit altitude, km

Thus, the required performance for the direct ascent portion of the mission is reduced to a total vacuum-equivalent ΔV of

3. $\Delta V_{tot} = (V_{circ} + \Delta V_{pen} - V_{rot})$,

where V_{rot} is the contribution due to the Earth's rotation. For retrograde orbits, the latter will be a penalty.

The vacuum-equivalent delta-V of the launch vehicle can be determined by applying the rocket equation to the launcher stage(s) used during this portion of the mission - remembering that a portion of the upper-stage propellant may be reserved for on-orbit maneuvers, bookmarked as payload and unavailable for use here. Vacuum performance figures should be used for all engines, even those used primarily at low altitude. Parallel staging (e.g. strap-on boosters) will complicate this somewhat, but the calculation is still relatively simple.

Concurrent with this calculation, one must calculate the time required for the direct-ascent phase of the flight. This will for the most part be a simple burn time calculation, though for best results staging and ignition delays should be considered. If a vehicle configuration allows for an optional coast before upper-stage ignition, the most accurate estimate generally comes from using the minimum coast period.

With the ascent time and parking orbit altitude known, equation 1 gives the total estimated penalty delta-V. If the parking orbit altitude is not known, a value of 100 nm or 185 km will serve as a reasonably accurate estimate for most modern launch vehicles. If the penalty delta-V satisfies equation 3, the launcher has sufficient performance for the proposed mission. If necessary, the calculation may be repeated with a new payload estimate, iterating until closure.

The Townsend approximation for the penalty term assumes that all of the various penalty effects can be reduced to a simple function of the ascent time, which seems deceptively simple. However, most of the "penalty" relates to events occurring early in the flight, e.g. drag and gravity losses. The chief constraint on a vehicle designer's effort to overcome these losses, is the time spent in this regime. If the vehicle flies a relatively uniform acceleration profile, total ascent time will correlate strongly with the time spent in the near-vertical, high-drag regime where most loss effects are found.

One further source of 'penalty' delta-V, is trajectory shaping losses. These are most often due to a requirement that the early trajectory loft the vehicle to a sufficiently high altitude, sometimes higher than the parking orbit altitude, that it will not fall back into the atmosphere before reaching orbital velocity. This is even more strongly correlated with total ascent time; a longer ascent period requires greater excess vertical velocity.

However, Townsend's work explicitly assumed multistage vehicles with identical stage mass ratios, T:W ratios, and specific impulses. Given that assumption, the correlation between total direct-ascent time and early flight behavior will be quite good. The resulting launch vehicle, will not be so good, and will not reflect modern design practice. If dissimilar staging is allowed, the optimal design is generally one which uses a high-thrust first stage (often augmented by strap-on boosters) to rapidly escape the early high-loss flight regime, and a relatively low thrust but high Isp upper stage to most efficiently add delta-V once most loss terms have become irrelevant.

Thus, for example, an Ariane 5G launch vehicle requires an astonishingly long 1,500 seconds to fly a direct ascent LEO mission, in spite of a very high 1.9:1 initial thrust:weight ratio. Clearly, for such a vehicle the total ascent time is not a good proxy for early flight behavior. The Ariane 5, and many other modern launch vehicles, accelerates briskly through the high-loss regime, then almost leisurely works its way up to orbital velocity. And its behavior is very, very poorly predicted by this model in its basic form.

A simple refinement, however, restores the utility of the model. Instead of using the absolute time required to accelerate to local circular orbit velocity, we can use the weighted average of A: the true acceleration time, and B: the acceleration time of a hypothetical three-stage-to-orbit vehicle where each stage has the same mass ratio, and the vacuum I_{sp} and initial T:W ratio of the actual vehicle at launch. Such a vehicle will behave quite similar to the actual vehicle in the critical early period of the trajectory, but will more accurately match the underlying assumptions of the Townsend Model.

The equivalent ascent time of the hypothetical three-stage vehicle is,

$$4.\ T_{3s} = 3\ [\ 1 - e^{(-0.333\ \Delta Vp\ /\ g\ Isp)}\ ]\ g\ I_{sp}\ /A_0$$

where ΔV_p = delta-V to parking orbit, m/s
A_0 = acceleration at launch, neglecting gravity and atmospheric effects, m/s^2

Based on a survey of over 1,000 data points representing the known performance of 17 modern launch vehicles to a variety of destination orbits, the best-fit modification of equation 2 becomes:

5. $\Delta V_{pen} = K_3 + K_4 T_{mix}$

where $K_3 = 429.9 + 1.602\ H_p + 1.224E\text{-}3\ H_p^2$
$K_4 = 2.328 - 9.687E\text{-}4\ H_p$
H_p = Parking orbit altitude, km

where,

6. $T_{mix} = 0.405\ T_a + 0.595\ T_{3s}$

This model predicts the effective penalty delta-V with an RMS error of ~260 m/s. With the total delta-V of a space launch mission ranging from ~8,500 m/s (ideal LEO launch) to 13,000 m/s or more (Earth escape or GEO payload delivery), this gives a <3% error in total mission delta-V and usually <10% error in payload capacity.

It is possible to do better still, for launchers whose performance is known in at least some cases. Comparing known performance with that predicted by the model, one often sees a systematic offset that can then be reasonably applied to the same launcher in a new configuration or application. In some cases, offsets can be identified for specific components of a modular launcher.

For example, an examination of 210 data points reflecting the performance of the Delta IV launch vehicle family, most of them from the Boeing Delta IV payload planner's guide, shows generally good agreement with the model. The accuracy is greatly increased, with an RMS error of only 113 m/s in mission delta-V, if we add an offset of +314.8 m/s to the core vehicle, -70.5 m/s to each GEM-60 solid rocket booster, and 119.4 m/s to the optional 5-meter payload fairing. This suggests that the Delta IV family was designed for optimal performance with the full compliment of boosters, and is somewhat less efficient in with just the core vehicle. Also, unsurprisingly, the larger payload fairing has a non-trivial performance penalty due to increased drag.

With this additional knowledge, it is possible to very accurately predict the performance of the Delta IV to orbits not listed in the payload planner's guide, or to examine the impact of potential changes to the vehicle configuration (e.g. a twin-engine upper stage)

Finally, it is worth noting some of the limitations of this technique. It requires only basic engineering data on the rocket stages in use, but that data must be accurate - the survey of current launchers conducted during this work, revealed several cases where published engineering and/or performance data was clearly inaccurate; often because bare motor weights were quoted for upper stages that in fact incorporate substantial additional hardware.

It is also necessary to accurately model the orbital maneuvers conducted in the final stage of a payload launch. These have not been discussed here, and for the most part represent a simple problem in orbital mechanics - especially for launches to circular orbits in a plane directly accessible from the launch site, where a near-impulsive Hohmann transfer can be performed. As previously mentioned, however, there may be operational constraints on these orbital maneuvers which must be taken into account.

It is possible to "cheat" and predict unrealistically high performance, by positing strap-on boosters or even entire first stages which provide high thrust for an extremely short period. This would extrapolate to a very short burn time per equation 6, but the expected performance gains will not materialize if the vehicle reverts to a low-thrust trajectory too early in the flight. Great skepticism should be exercised regarding any vehicle whose first stage and/or SRBs burn for significantly less than one minute.

A related issue concerns fast-burn boosters, especially those derived from surplus ICBMs. The original Townsend model shows net delta-V losses reducing monotonically with ascent time, down to a period of two hundred seconds. The more complete survey here confirms this, and suggests the extrapolation might continue down to perhaps 100-150 seconds. But a sufficiently high-acceleration launcher will reach impractically high velocities while still in the lower atmosphere, resulting in excessive drag losses. This is not considered in the present model, and is a potential source of error.

And, obviously, this is a technique which in its present form is limited to ground-launched, ballistic, rocket vehicles. An extension to air-launched vehicles is possible, and may be implemented in the future. But vehicles which use air-breathing propulsion, or which make significant use of aerodynamic lift during the ascent phase, will use substantially different trajectories and suffer very different penalties - at present, there seems no alternative but a comprehensive 3DOF trajectory simulation for such vehicles.